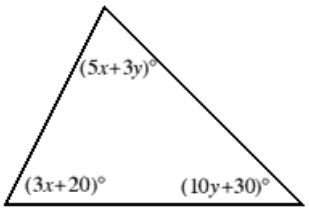
JMC Number Theory

1. [JMC 2012 Q23] Peter wrote a list of all the numbers that could be produced by changing one digit of the number 200. How many of the numbers on Peter's list are prime?

   A 0 B 1 C 2 D 3 E 4

2. [JMC 2012 Q25] The interior angles of a triangle are $(5x + 3y)°$, $(3x + 20)°$ and $(10y + 30)°$ where x, y are positive integers.
   What is the value of $x + y$?



   A 15 B 14 C 13 D 12 E 11

3. [JMC 2011 Q19] A list is made of every digit that is the units digit of at least one prime number. How many of the following numbers appear in the list?

   A 1 B 2 C 3 D 4 E 5

4. [JMC 2011 Q22] Last week Evariste and Sophie both bought some stamps for their collections. Each stamp Evariste bought cost him £1.10, whilst Sophie paid 70p for each of her stamps. Between them they spent exactly £10. How many stamps did they buy in total?

   A 9 B 10 C 11 D 12 E 13

5. [JMC 2010 Q18] Sam's 101st birthday is tomorrow. So Sam's age in years changes from a square number (100) to a prime number (101). How many times has this happened before in Sam's lifetime?

   A 1 B 2 C 3 D 4 E 5

6. [JMC 2010 Q22] Kiran writes down six different prime numbers, p, q, r, s, t, u, all less than 20, such that $p + q = r + s = t + u$. What is the value of $p + q$?

   A 16 B 18 C 20 D 22 E 24

7. [JMC 2010 Q24] The year 2010 belongs to a special sequence of twenty-five consecutive years: each number from 1988 to 2012 contains a repeated digit.
   Each of the following belongs to a sequence of consecutive years, where each number in the sequence contains at least one repeated digit.
   Which of them belongs to the next such sequence of at least twenty years?

A 2099 B 2120 C 2199 D 2989 E 3299

8. [JMC 2010 Q25] What is the value of $P + Q + R$ in the multiplication on the right?

$$\begin{array}{cccccc} & P & Q & P & Q \\ \times & & R & R & R \\ \hline 6 & 3 & 9 & 0 & 2 & 7 \\ \hline \end{array}$$

A 13 B 12 C 11 D 10 E 9

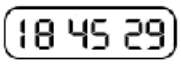
9. [JMC 2009 Q25] In Miss Quaffley's class, one third of the pupils bring a teddy bear to school. Last term, each boy took 12 books out of the library, each girl took 17 books and each teddy bear took 9 books. In total, 305 books were taken out. How many girls are there in Miss Quaffley's class?

A 4 B 7 C 10 D 13 E 16

10. [JMC 2008 Q20] If all the whole numbers from 1 to 1000 inclusive are written down, which digit appears the smallest number of times?

A 0 B 2 C 5 D 9
E none: no single digit appears fewer times than all the others

11. [JMC 2008 Q22] On a digital clock displaying hours, minutes and seconds, how many times in each 24-hour period do all six digits change simultaneously?



A 0 B 1 C 2 D 3 E 24

12. [JMC 2008 Q24] The list $2, 1; 3, 2;\ 2, 3; 1, 4$ describes itself, since that are two 1s, three 2s, two 3s and one 4. There is exactly one other list of eight numbers containing only the numbers, 1, 2, 3 and 4 that, in the same way, describes the numbers of 1s, 2s, 3s and 4s, in that order. What is the total number of 1s and 3s in this other list?

A 2 B 3 C 4 D 5 E 6

13. [JMC 2007 Q18] The letters J, M, C represent three different non-zero digits. What is the value of $J + M + C$?

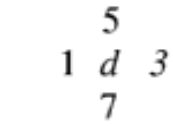
$$\begin{array}{cc} J & J \\ M & M \\ C & C \\ \hline J & M & C \\ \hline \end{array}$$

A 19 B 18 C 17 D 16 E 15

14. [JMC 2007 Q21] A list of ten numbers contains two of each of the numbers 0, 1, 2, 3, 4. The two 0s are next to each other, the two 1s are separated by one number, the two 2s by two numbers, the two 3s by three numbers and the two 4s by four numbers. The list starts 3, 4, ... What is the last number?

A 0 B 1 C 2 D 3 E 4

15. [JMC 2007 Q22] Only one choice of the digit d gives a prime number for each of the three-digit numbers read across and downwards in the diagram on the right. Which digit is d?



A 4 B 5 C 6 D 7 E 8

16. [JMC 2007 Q24] The pages of a book are numbered 1, 2, 3, ... In total, it takes 852 digits to number all the pages of the book. What is the number of the last page?

A 215 B 314 C 320 D 329 E 422

17. [JMC 2006 Q20] The sum of three different prime numbers is 40. What is the difference between the two biggest of these three numbers?

A 8 B 12 C 16 D 20 E 24

18. [JMC 2006 Q22] A positive whole number less than 100 has remainder 2 when it is divided by 3, remainder 3 when it is divided by 4 and remainder 5 when it is divided by 5. What is its remainder what it is divided by 7?

A 2 B 3 C 4 D 5 E 6

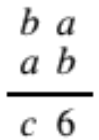
19. [JMC 2006 Q24] Amrita has written down four whole numbers. If she chooses three of her numbers at a time and adds up each triple, she obtains totals of 115, 153, 169 and 181. What is the largest of Amrita's numbers?

A 66 B 53 C 91 D 121 E 72

20. [JMC 2006 Q25] For how many positive values of n are both $\frac{1}{2}n$ and $2n$ three-digit whole numbers?

A 0 B 150 C 200 D 300 E 500

21. [JMC 2005 Q18] In the subtraction sum on the right a, b and c are digits, and a is less than b. What is the value of c?



A 3 B 4 C 5 D 6 E a number greater than 6

22. [JMC 2004 Q22] The digits in the product $13 \times 2 = 26$ can be rearranged to give $16 \times 2 = 32$ as well as $31 \times 2 = 62$. In which one of the following can the digits not be rearranged to give another correct product?

A $12 \times 3 = 36$ B $12 \times 7 = 84$ C $26 \times 3 = 78$
D $16 \times 3 = 48$ E $39 \times 2 = 78$

23. [JMC 2004 Q23] In this addition each letter stands for a different digit, with S standing for 3. What is the value of $Y \times O$?

$$\begin{array}{cccccc} & & & S & O \\ + & M & A & N & Y \\ \hline & S & U & M & S \end{array}$$

A 0 B 2 C 36 D 40 E 42

Answers

1. A
2. A
3. D
4. D
5. D
6. E
7. B
8. A
9. B
10. A
11. D
12. E
13. B
14. B
15. D
16. C
17. E
18. B
19. C
20. B
21. A
22. D
23. E